

Fock-Lorentz transformations and time-varying speed of light

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Abstract

The theory of relativity was built up on linear Lorentz transformation. However, in his fundamental work “Theory of Space, Time and Gravitation” [1] V.A.Fock shows that the general form of the transformation between the coordinates in the two inertial frames could be taken to be linear fractional. The implicit form of this transformation contains two constants of different space-time dimensions. They can be reduced to the constant c with the dimension of speed (“speed of light”), and to the constant R with the dimension of length (an invariant radius of the visible part of the Universe). The geometry of the “light cones” shows that R is a fundamental constant, but c depends on the time of transformation.

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I. Introduction

The Theory of Relativity was built up on two postulates, namely, the principle of relativity and another principle that states that the velocity of light is independent of the velocity of its source. Only the combination of the two principles requires the transformation formulae between the orthogonal rectilinear coordinates in the two inertial frames to be linear. However, from 1910 it was known [2], [3] that constancy of the speed of light is a consequence of the principle of relativity and general homogeneity properties of the space-time manifold in the case of linear transformations. Many authors [4]-[21] have postulated linear transformations or added different (but equivalent) hypotheses with the purpose to get the constancy of the speed of light as a consequence of relativity.

In his fundamental work “Theory of space, time and gravitation” [1] Fock shows that constancy of the speed of light is equivalent to the linearity of the transformations. In the same work in Appendix A it is shown, that from the principle of relativity alone one can get a more general *linear fractional* form of the transformation between two inertial frames (Fock-Lorentz or FL transformation below).

One can take any conventional method to construct reference frames (tool kit for space-time measurements). If the velocity of any body in system k is constant and equal to \mathbf{u} , we can introduce system k' rigidly linked to the moving body in such a way that the velocity of the origin of the frame k in k' is equal to $-\mathbf{u}$. The arbitrariness of the common scaling of length and time units can be eliminated by the symmetry of direct and inverse transformations (from k to k' and from k' to k) — those transformations must differ only by the sign of \mathbf{u} .

The usual *a priori* assumption that all bodies at rest in k' have the same velocity \mathbf{u} in k is equivalent to linearity of the transformation. This assumption is not valid in the case of linear fractional transformations. So, we had to be careful in the definition of the relative velocities. Let us introduce coordinates in such a way that point $\mathbf{r} = 0$ at rest in frame k has constant velocity \mathbf{u} in k' ; point $\mathbf{r}' = 0$ at rest in frame k' has constant velocity $-\mathbf{u}$ in k . We take that at $t = 0$ the spatial origins of both frames coincide and $t' = 0$.

Let us write the general transformation for transition from a system k to a system k' as ratios of linear functions, all with the same denominator:

$$t' = \frac{a_{tt}(u)t + a_{tx}(u)x + a_{ty}(u)y + a_{tz}(u)z}{A(u) + A_t(u)t + A_x(u)x + A_y(u)y + A_z(u)z}, \quad (1)$$

$$x' = \frac{a_{xt}(u)t + a_{xx}(u)x + a_{xy}(u)y + a_{xz}(u)z}{A(u) + A_t(u)t + A_x(u)x + A_y(u)y + A_z(u)z}, \quad (2)$$

$$y' = \frac{a_{yt}(u)t + a_{yx}(u)x + a_{yy}(u)y + a_{yz}(u)z}{A(u) + A_t(u)t + A_x(u)x + A_y(u)y + A_z(u)z}, \quad (3)$$

$$z' = \frac{a_{zt}(u)t + a_{zx}(u)x + a_{zy}(u)y + a_{zz}(u)z}{A(u) + A_t(u)t + A_x(u)x + A_y(u)y + A_z(u)z}. \quad (4)$$

Here u is the relative velocity, $a_{\mu\nu}(u)$, $A_\mu(u)$ and $A(u)$ are arbitrary functions of the relative velocity. The denominator in Eqs.(1)-(4) can go to zero. If we require the finiteness of coordinates of all points in all inertial systems, we simplify Eqs.(1)-(4) down to the usual linear transformation: $A(u) = 1$, all $A_\mu(u) = 0$. This requirement seems natural for uniform Euclidean space. However, it became clear from Friedmann's work [22] that our space-time can have a more complex structure. Existence of the island masses can lead to space anomalies, uniform mass distribution across the whole space leads to the big-bang anomalies. Eqs.(1)-(4) operates in such space-time which can serve as a background for the real Universe scenario.

In Section II we present the derivation of the explicit form of the FL transformation.

Section III is devoted to some geometrical consequences of this transformation. Main of them is the relativity of infinity: points which are infinitely distant across space-time in one reference system appear to be on a finite distance when viewed from another system. Under the FL transformation, infinity no longer can be considered as an invariant point. When we are going from Galileo to Lorentz transformations, the infinite invariant velocity becomes finite invariant velocity c . Now, when we are going from Lorentz to Fock-Lorentz transformations, the infinitely large invariant space-time distance becomes finite invariant space-time distance.

A new fundamental constant R appears in the FL transformation. For distances $L \ll R$ and times $T \ll R/c$, FL transformation becomes the usual Lorentz transformation. So it is clear that R should stand for some cosmological space distance.

In Section IV the velocity and the energy-momentum four-vectors are constructed. They appear to be Lorentz-covariant under the FL transformation of the space-time manifold.

In Section V we study the geometry of FL-covariant world lines of photons, bradions

and tachions and get an explicit speed of light dependence on time: $c(t) = R/t$, where $t = 0$ in the peculiar point of the transformation in linear fractions. In this point all bradions are concentrated into the invariant sphere of radius R , and all photons — on the surface of this sphere.

In works [23]-[31], a Varying Speed of Light (VSL) model is discussed as a possible alternative to inflationary cosmology. This model may resolve some cosmological puzzles, but the authors usually postulate breaking Lorentz invariance of space-time or of fundamental interactions.

In our approach, Lorentz invariance is local but Fock-Lorentz invariance and varying speed of light are global fundamental properties of space-time.

In Section VI we show the connection between the FL transformation and the Friedmann-Lobachevsky space geometry.

In Section VII we present some formulae for the red shift of expanding galaxies and for the usual longitudinal Doppler effect in the VSL model. A non-trivial connection between the Hubble constant H_1 and the red-shift parameter z is established (we write H_0 for conventional Hubble constant cz/r).

In Appendix I we explain the mathematical origin of the linear fractional transformation with common denominator.

II. Derivation of the explicit form of the Fock-Lorentz transformation

We can rewrite Eqs.(1)-(4) in vector form and take into account the isotropy of our space:

$$t' = \frac{a(u)t + b(u)(\mathbf{u}\mathbf{r})}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}, \quad (5)$$

$$\mathbf{r}'_{||} = \frac{d(u)\mathbf{u}t + e(u)\mathbf{r}_{||}}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}, \quad (6)$$

$$\mathbf{r}'_{\perp} = \frac{f(u)\mathbf{r}_{\perp}}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}. \quad (7)$$

Here $a(u)$, $b(u)$, $d(u)$, $e(u)$, $f(u)$, $A(u)$, $B(u)$, $D(u)$ are unknown functions of the relative velocity u .

Now we fix these functions.

1. Transformation (5)-(7) will not be changed after replacement of $\mathbf{r}_{||}$ for $-\mathbf{r}_{||}$, $\mathbf{r}'_{||}$ for $-\mathbf{r}'_{||}$, \mathbf{u} for $-\mathbf{u}$, since such replacement is equivalent to the rotation around \mathbf{r}_{\perp} by the angle π .

Corollary: all unknown functions in Eqs.(5)-(7) are simultaneously odd or simultaneously even. It is natural to choose even functions.

2. Point $\mathbf{r}' = 0$ has velocity \mathbf{u} in frame k :

$$d(u) = -e(u)$$

3. Point $\mathbf{r} = 0$ has velocity $-\mathbf{u}$ in frame k' :

$$a(u) = -d(u) = e(u).$$

We can substitute

$$b(u) = -a(u)g(u)$$

and get

$$t' = \frac{a(u)(t - g(u)(\mathbf{u}\mathbf{r}))}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}, \quad (8)$$

$$\mathbf{r}'_{||} = \frac{a(u)(\mathbf{r}_{||} - \mathbf{u}t)}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}, \quad (9)$$

$$\mathbf{r}'_{\perp} = \frac{f(u)\mathbf{r}_{\perp}}{A(u) + B(u)t + D(u)(\mathbf{u}\mathbf{r})}. \quad (10)$$

4. The inverse transformation from k' to k must be the same as in Eqs.(8)-(10) with $-\mathbf{u}$ instead of \mathbf{u} :

$$\begin{aligned} t &= \frac{a(u)(t' + g(u)(\mathbf{u}\mathbf{r}'))}{A(u) + B(u)t' - D(u)(\mathbf{u}\mathbf{r}')} \\ \mathbf{r}_{||} &= \frac{a(u)(\mathbf{r}'_{||} + \mathbf{u}t')}{A(u) + B(u)t' - D(u)(\mathbf{u}\mathbf{r}')} \\ \mathbf{r}_{\perp} &= \frac{f(u)\mathbf{r}'_{\perp}}{A(u) + B(u)t' - D(u)(\mathbf{u}\mathbf{r}')}. \end{aligned}$$

Combination of direct and inverse transformations will reduce to the identity, which means that

$$\begin{aligned} A(u) &= a(u)\sqrt{1 - g(u)u^2}, \\ B(u) &= -\frac{D(u)}{g(u)}\sqrt{1 - g(u)u^2}, \\ f(u) &= A(u). \end{aligned}$$

We can introduce

$$D(u) = h(u)A(u)$$

$$\sqrt{1 - g(u)u^2} = \gamma^{-1}(u),$$

and get

$$t' = \frac{\gamma(u)(t - g(u)(\mathbf{u}\mathbf{r}))}{1 + \frac{h(u)}{g(u)}(1 - \gamma^{-1}(u))t + h(u)(\mathbf{u}\mathbf{r})}, \quad (11)$$

$$\mathbf{r}'_{||} = \frac{\gamma(u)(\mathbf{r}_{||} - \mathbf{u}t)}{1 + \frac{h(u)}{g(u)}(1 - \gamma^{-1}(u))t + h(u)(\mathbf{u}\mathbf{r})}, \quad (12)$$

$$\mathbf{r}'_{\perp} = \frac{\mathbf{r}_{\perp}}{1 + \frac{h(u)}{g(u)}(1 - \gamma^{-1}(u))t + h(u)(\mathbf{u}\mathbf{r})}. \quad (13)$$

5. Now we write transformation from k' to other frame k'' , which is moving with velocity \mathbf{u}' in k' . (We can put $\mathbf{u}'||\mathbf{u}$ without any lack of generality):

$$t'' = \frac{\gamma(u')(t' - g(u')(\mathbf{u}'\mathbf{r}'))}{1 + \frac{h(u')}{g(u')}(1 - \gamma^{-1}(u'))t' + h(u')(\mathbf{u}'\mathbf{r}'))}, \quad (14)$$

$$\mathbf{r}''_{||} = \frac{\gamma(u')(\mathbf{r}'_{||} - \mathbf{u}'t')}{1 + \frac{h(u')}{g(u')}(1 - \gamma^{-1}(u'))t' + h(u')(\mathbf{u}'\mathbf{r}'))}, \quad (15)$$

$$\mathbf{r}''_{\perp} = \frac{\mathbf{r}'_{\perp}}{1 + \frac{h(u')}{g(u')}(1 - \gamma^{-1}(u'))t' + h(u')(\mathbf{u}'\mathbf{r}'))}, \quad (16)$$

Combine Eqs.(11)-(13) and (14)-(16), and get the transformation from k to k'' with some new relative velocity u'' . Such transformation,

$$t'' = \frac{\gamma(u'')(t - g(u'')(\mathbf{u}''\mathbf{r}))}{1 + \frac{h(u'')}{g(u'')}(1 - \gamma^{-1}(u''))t + h(u'')(\mathbf{u}''\mathbf{r})},$$

$$\mathbf{r}''_{||} = \frac{\gamma(u'')(\mathbf{r}_{||} - \mathbf{u}''t)}{1 + \frac{h(u'')}{g(u'')}(1 - \gamma^{-1}(u''))t + h(u'')(\mathbf{u}''\mathbf{r})},$$

$$\mathbf{r}''_{\perp} = \frac{\mathbf{r}_{\perp}}{1 + \frac{h(u'')}{g(u'')}(1 - \gamma^{-1}(u''))t + h(u'')(\mathbf{u}''\mathbf{r})},$$

will take place only if

$$1. \quad g(u) = g(u') = g(u'') \equiv \frac{1}{c^2},$$

$$\begin{aligned}
2. \quad u'' &= \frac{u + u'}{1 + \frac{uu'}{c^2}}, \\
3. \quad h(u) &= \gamma(u) \frac{1}{Rc}.
\end{aligned}$$

Here c is constant with the dimension of velocity, R is constant with the dimension of length.

At last the final form of FL transformation is

$$t' = \frac{\gamma(u) (t - \mathbf{u}\mathbf{r}/c^2)}{1 - (\gamma(u) - 1) ct/R + \gamma(u)\mathbf{u}\mathbf{r}/Rc}, \quad (17)$$

$$\mathbf{r}'_{||} = \frac{\gamma(u)(\mathbf{r}_{||} - \mathbf{u}t)}{1 - (\gamma(u) - 1) ct/R + \gamma(u)\mathbf{u}\mathbf{r}/Rc}, \quad (18)$$

$$\mathbf{r}'_{\perp} = \frac{\mathbf{r}_{\perp}}{1 - (\gamma(u) - 1) ct/R + \gamma(u)\mathbf{u}\mathbf{r}/Rc}, \quad (19)$$

In the limit $r \ll |R|$ and $ct \ll |R|$, Eqs.(17)-(19) coincide with the usual Lorentz transformation.

It is easy to observe that Eqs.(17)-(19) are equivalent to the Lorentz transformation

$$\frac{t'}{1 + ct'/R} = \frac{\gamma(u) (t - \mathbf{u}\mathbf{r}/c^2)}{1 + ct/R}, \quad (20)$$

$$\frac{\mathbf{r}'_{||}}{1 + ct'/R} = \frac{\gamma(u)(\mathbf{r}_{||} - \mathbf{u}t)}{1 + ct/R}, \quad (21)$$

$$\frac{\mathbf{r}'_{\perp}}{1 + ct'/R} = \frac{\mathbf{r}_{\perp}}{1 + ct/R}, \quad (22)$$

for the quantities

$$\tilde{t} = \frac{t}{1 + ct/R}, \quad \tilde{\mathbf{r}} = \frac{\mathbf{r}}{1 + ct/R}. \quad (23)$$

III. Properties of the FL transformation

From Eqs.(20)-(23) one can construct the finite invariant

$$c^2\tilde{t}^2 - \tilde{\mathbf{r}}^2 = \frac{c^2t^2 - \mathbf{r}^2}{(1 + ct/R)^2} = \text{inv} \quad (24)$$

and the line element

$$ds^2 = c^2d\tilde{t}^2 - d\tilde{\mathbf{r}}^2 = \frac{(1 - r^2/R^2)c^2dt^2 - (1 + ct/R)^2d\mathbf{r}^2 + 2cdt(1 + ct/R)\mathbf{r}d\mathbf{r}/R}{(1 + ct/R)^4}. \quad (25)$$

Let us find spacelike and timelike “hyperboloids”. For positive sign of invariant (24) one can get for any world point

$$\frac{c^2 t^2 - \mathbf{r}^2}{(1 + ct/R)^2} = \frac{c^2 \tau^2}{(1 + c\tau/R)^2} \quad (26)$$

where τ is a “proper time” — time in the space origin of the rest frame.

For $\tau > 0$, “hyperboloid” (26) has asymptote $r \rightarrow \infty$

$$ct_{as} = \pm r \frac{1 + c\tau/R}{\sqrt{1 + 2c\tau/R}} + \frac{c^2 \tau^2/R}{1 + 2c\tau/R}. \quad (27)$$

This “hyperboloid” has two parts — one in the region $t > \tau$ and the other in the region $t < -R/c$.

All time-like “hyperboloids” have common S_3^- intersection ($ct = -R$, $\mathbf{r}^2 = R^2$) of the light cone $c^2 t^2 = \mathbf{r}^2$ with the hyperplane $ct = -R$.

For negative τ the entire “hyperboloid” is inside the light cone of the past and includes the same S_3^- .

For negative values of interval (24) we can write

$$\frac{c^2 t^2 - \mathbf{r}^2}{(1 + ct/R)^2} = -\lambda^2. \quad (28)$$

Here λ is a space coordinate of the world point in the frame, where its time coordinate is $t = 0$. All space-like “hyperboloids” are tangent to the light cone $c^2 t^2 = \mathbf{r}^2$ on the same S_3^- manifold.

Asymptotes for $r \rightarrow \infty$ are

$$ct_{as} = \pm r \frac{1}{\sqrt{1 + \lambda^2/R^2}} - \frac{\lambda^2/R}{1 + \lambda^2/R^2}. \quad (29)$$

Assume that some particle was produced at time $t = 0$ and decayed at time τ in its own rest frame. In another frame of reference, where this particle had velocity \mathbf{u} , it decayed at time t in a point with coordinate $\mathbf{r} = \mathbf{u}t$. From Eq.(24) we can get its lifetime in the “moving” frame as

$$t = \frac{\tau}{(1 + c\tau/R)\sqrt{1 - \mathbf{u}^2/c^2} - c\tau/R}. \quad (30)$$

Some apparent properties of Eq.(30):

A. If the velocity of the particle in any frame is sufficiently great

$$u > \frac{\sqrt{1 + 2c\tau/R}}{1 + c\tau/R} \quad (31)$$

or, in other words, if its proper lifetime is large enough

$$\tau > \frac{R/c}{\gamma(u) - 1},$$

we get from Eq.(30) that the world line of this particle is going from the origin till infinity in the future light cone and then from infinity till $t < 0$ in the past light cone. So this particle is seen to be “stable” in frames (31).

B. For $\tau = -R/c$ we get from Eq.(30) that this time interval is the same ($t = -R/c$) in all inertial frames of reference. So, in our approach the special finite time interval $-R/c$ is invariant, but infinite time intervals are not.

If any body is at rest in any frame of reference at distance \mathbf{r} from the origin, then in another frame, moving with velocity \mathbf{u} , this body will be at distance \mathbf{r}'_0 from the origin at time $t' = 0$. Let us find \mathbf{r}'_0 . Put $t' = 0$, $\mathbf{r}' = \mathbf{r}'_0$ in Eqs.(17)-(19), eliminate t and get

$$\begin{aligned} \mathbf{r}'_{0\parallel} &= \frac{\gamma^{-1}(u)\mathbf{r}_{\parallel}}{1 + \mathbf{u}\mathbf{r}/(Rc)}, \\ \mathbf{r}'_{0\perp} &= \frac{\mathbf{r}_{\perp}}{1 + \mathbf{u}\mathbf{r}/(Rc)}. \end{aligned}$$

Let us take a small body, which is moving in frame k with velocity \mathbf{u} . Space-time coordinates of this body are (t, \mathbf{r}) . In another frame k' , which is moving with velocity \mathbf{v} in k , this body has the velocity \mathbf{u}' . Let us find \mathbf{u}' . At first we can rewrite Eqs.(20)-(22) for space-time differentials:

$$\frac{dt'}{(1 + ct'/R)^2} = \gamma(v) \frac{dt (1 + \mathbf{v}\mathbf{r}/(Rc)) - (1 + ct/R)\mathbf{v}d\mathbf{r}/c^2}{(1 + ct/R)^2}, \quad (32)$$

$$\frac{d\mathbf{r}'_{\parallel}(1 + ct'/R) - \mathbf{r}_{\parallel}cdt'/R}{(1 + ct'/R)^2} = \gamma(v) \frac{d\mathbf{r}_{\parallel}(1 + ct/R) - cdt(\mathbf{v} + c\mathbf{r}_{\parallel}/R)}{(1 + ct/R)^2}, \quad (33)$$

$$\frac{d\mathbf{r}'_{\perp}(1 + ct'/R) - \mathbf{r}'_{\perp}cdt'/R}{(1 + ct'/R)^2} = \frac{d\mathbf{r}_{\perp}(1 + ct/R) - \mathbf{r}_{\perp}cdt/R}{(1 + ct/R)^2}. \quad (34)$$

Now we introduce auxillary quantities $\mathbf{r}_0 = \mathbf{r} - \mathbf{u}t$, and $\mathbf{r}'_0 = \mathbf{r}' - \mathbf{u}'t'$, and divide Eqs. (33), (34) by Eq.(32):

$$\mathbf{u}'_{\parallel} - c\mathbf{r}'_{0\parallel}/R = \frac{\mathbf{u}_{\parallel} - c\mathbf{r}_{0\parallel}/R - \mathbf{v}}{1 - \mathbf{v}(\mathbf{u} - c\mathbf{r}_0/R)/c^2}, \quad (35)$$

$$\mathbf{u}'_{\perp} - c\mathbf{r}'_{0\perp}/R = \gamma^{-1}(v) \frac{\mathbf{u}_{\perp} - c\mathbf{r}_{0\perp}/R}{1 - \mathbf{v}(\mathbf{u} - c\mathbf{r}_0/R)/c^2}. \quad (36)$$

The latter Eqs.(35), (36) are appropriate for the investigation of the FL transformation properties, but are not convenient for practical use. The left hand sides of Eqs.(35), (36) contain \mathbf{u}' not only explicitly, but also through \mathbf{r}'_0 . Let us connect \mathbf{r}'_0 and \mathbf{r}_0 . If we rewrite Eqs.(20)-(22) for the definite world point $t' = 0$, $\mathbf{r}' = \mathbf{r}'_0$, we get

$$0 = \frac{\gamma(v)(t - \mathbf{v}\mathbf{r}/c^2)}{1 + ct/R}, \quad (37)$$

$$\mathbf{r}'_{0||} = \frac{\gamma(v)(\mathbf{r}_{||} - \mathbf{v}t)}{1 + ct/R}, \quad (38)$$

$$\mathbf{r}'_{0\perp} = \frac{\mathbf{r}_{\perp}}{1 + ct/R}. \quad (39)$$

From Eq.(37) and definiton $\mathbf{r}_0 = \mathbf{r} - \mathbf{u}t$, we get for this world point

$$\mathbf{r}_0 = \mathbf{r} - \mathbf{u}(\mathbf{v}\mathbf{r})/c^2. \quad (40)$$

The inverse to Eq.(40) are

$$\begin{aligned} \mathbf{r}_{||} &= \frac{\mathbf{r}_{0||}}{1 - \mathbf{v}\mathbf{u}/c^2}, \\ \mathbf{r}_{\perp} &= \frac{\mathbf{r}_{0\perp} + [\mathbf{v}[\mathbf{u}\mathbf{r}_0]]/c^2}{1 - \mathbf{v}\mathbf{u}/c^2}. \end{aligned}$$

Now we can put Eqs.(38)-(39), in right-hand parts of Eqs.(38), (39):

$$\begin{aligned} \mathbf{r}'_{0||} &= \gamma^{-1}(v) \frac{\mathbf{r}_{0||}}{1 - \mathbf{v}(\mathbf{u}/c^2 - \mathbf{r}_0/(Rc))}, \\ \mathbf{r}'_{0\perp} &= \frac{\mathbf{r}_{0\perp} + [\mathbf{v}[\mathbf{u}\mathbf{r}_0]]/c^2}{1 - \mathbf{v}(\mathbf{u}/c^2 - \mathbf{r}_0/(Rc))} \end{aligned}$$

and after some dull calculations rewrite Eqs.(35)-(36) in manifest form

$$\mathbf{u}'_{||} = \frac{\mathbf{u}_{||} - \mathbf{v} - (1 - \sqrt{1 - v^2/c^2}) c(\mathbf{r}_{||} - \mathbf{u}_{||}t)/R}{1 - \mathbf{v}(\mathbf{u} - c(\mathbf{r} - \mathbf{u}t)/R)/c^2}, \quad (41)$$

$$\mathbf{u}'_{\perp} = \frac{\sqrt{1 - v^2/c^2}(\mathbf{u}_{\perp} - c(\mathbf{r}_{\perp} - \mathbf{u}_{\perp}t)/R) + c(\mathbf{r}_{\perp} - \mathbf{u}_{\perp}t + [\mathbf{v}[\mathbf{u}\mathbf{r}]]/c^2)/R}{1 - \mathbf{v}(\mathbf{u} - c(\mathbf{r} - \mathbf{u}t)/R)/c^2}. \quad (42)$$

Inspite of cumbersome appearance, the latter equations are suitable for direct calculations — the velocity \mathbf{u} in world point ct , \mathbf{r} (in frame k) is connected directly with the velocity \mathbf{u}' in frame k' in the same world point.

On the cones $\mathbf{r} = \mathbf{u}t$ ($\mathbf{r}_0 = 0$) we get the ordinary Einstein law for addition of velocities

$$\mathbf{u}'_{||} = \frac{\mathbf{u}_{||} - \mathbf{v}}{1 - \mathbf{v}\mathbf{u}/c^2}, \quad (43)$$

$$\mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\perp}\sqrt{1 - v^2/c^2}}{1 - \mathbf{v}\mathbf{u}/c^2}. \quad (44)$$

Let us return to the law for addition of velocities in the form of (35), (36). If in any inertial frame k some bodies (galaxies below) are moving at time $t = 0$ with velocities proportional to the distance from the origin $\mathbf{u} = c\mathbf{r}_0/R = H_0\mathbf{r}_0$, we get in the frame k' $\mathbf{u}' = c\mathbf{r}'_0/R - \mathbf{v}$. Transferring the origin into the other galaxy by the transformation $\mathbf{r}'_0 \rightarrow \mathbf{r}'_0 + R\mathbf{v}/c$ gives us $\mathbf{u}' = c\mathbf{r}'_0/R$. It means that the Hubble law of expansion ($R > 0$) or compression ($R < 0$) of the Universe with the constant $H_0 = c/R$ is the same for observers in all galaxies.

If any galaxy A has velocity $\mathbf{u}_A = c\mathbf{r}_{0A}/R$ in our frame, we can choose \mathbf{v} in Eq.(41) in such a way that

$$\gamma(v)\mathbf{v} = \mathbf{u}_A$$

and get the rest frame of galaxy A : $\mathbf{u}'_A = 0$. The transformation parameter

$$\mathbf{v} = \frac{\mathbf{u}_A}{\sqrt{1 + \mathbf{u}_A^2/c^2}} < c.$$

is meaningful for all values of the velocity \mathbf{u}_A , even for $\mathbf{u}_A > c$!

The experimental value¹ of H_0 is $H_0 = 100 \cdot h_0$ km/c/Mpc, with $0.6 < h_0 < 0.8$, which means that FL transformation parameter $R = \frac{1}{h_0}3 \cdot 10^3$ Mpc.

In some works [32]-[34] an original approach to the theory of relativity was introduced with universal constant H_0 . In our approach it is possible, in general, to choose any pair from R, c, H as universal constants. But, as it will be seen in Section V, the speed of light at the instant t from the special point of FL transformation is equal to $c = R/t$. So, R is really a fundamental constant, but $c = R/t$ and $H = 1/t$ are in some sense “time-measurement” quantities.

¹It will be shown in Section VII that the real value of the Hubble constant H_1 may be, less than H_0 in our approach.

IV. Energy-momentum four-vector

Let us write the metric line element (25) as

$$ds^2 = \frac{c^2 dt^2 \left(1 - (\mathbf{v}/c + \mathbf{v}t/R - \mathbf{r}/R)^2\right)}{(1 + ct/R)^4}. \quad (45)$$

or, for $ds^2 > 0$, as

$$ds = cdt \frac{\sqrt{1 - (\mathbf{v}/c - \mathbf{r}_{0p}/R)^2}}{(1 + ct/R)^2}, \quad (46)$$

where $\mathbf{v} = d\mathbf{r}/dt$ is the instantaneous velocity of some particle, $\mathbf{r}_{0p} = \mathbf{r} - \mathbf{v}t$ is an auxiliary “initial” space coordinate of the particle (the coordinate of the particle at the instant $t = 0$ in the case of a constant velocity \mathbf{v}).

Now we can introduce the four-vector

$$dx^\mu = \frac{1}{(1 + ct/R)^2} (cdt, d\mathbf{r}(1 + ct/R) - \mathbf{r}cdt/R), \quad (47)$$

which is constructed from the differentials of the Eq.(23) and undergoes the usual Lorentz transformation during the FL transformation of space-time. From Eqs.(46), (47) it is possible to construct the velocity four-vector:

$$u^\mu = \frac{dx^\mu}{ds} = \frac{1}{\sqrt{1 - (\mathbf{v}/c - \mathbf{r}_{0p}/R)^2}} (1, \mathbf{v}/c - \mathbf{r}_{0p}/R), \quad (48)$$

and the energy-momentum four-vector

$$p^\mu = mu^\mu = \frac{m}{\sqrt{1 - (\mathbf{v}/c - \mathbf{r}_{0p}/R)^2}} (1, \mathbf{v}/c - \mathbf{r}_{0p}/R). \quad (49)$$

We shall name the quantities in the right-hand side of Eq.(49) as energy

$$E = \frac{m}{\sqrt{1 - (\mathbf{v}/c - \mathbf{r}_{0p}/R)^2}} \quad (50)$$

and linear momentum

$$\mathbf{p} = \frac{m(\mathbf{v}/c - \mathbf{r}_{0p}/R)}{\sqrt{1 - (\mathbf{v}/c - \mathbf{r}_{0p}/R)^2}} \quad (51)$$

of the particle with mass m . The four-vectors of Eqs.(48), (49) transform like usual Lorentz vectors.

The energy and momentum get minimum values $E_0 = m$ and $\mathbf{p}_0 = 0$ not on the manifold of the rest frames (with $\mathbf{v} = 0$), connected by translations, but on the manifold of the “Hubble” frames (with $\mathbf{v} = c\mathbf{r}_{0p}/R$).

FL transformations introduce corrections to the nonrelativistic dynamics ($|\mathbf{v}| \ll c$), which are significant at large distances $|\mathbf{r}_{0p}| \approx R|v|/c$. For the velocity 1 m/s this distance is near 30 l.y.

Corrections to the relativistic dynamics are significant at extremely large distances $|\mathbf{r}_{0p}| \approx R$ only. Nevertheless, we see from Eqs.(50), (51) that even for $|\mathbf{r}_{0p}| \ll R$ a particle can move “faster” than light² if $c < |\mathbf{v}| < c(1 + r_{0p}/R)$.

V. Photons, bradyons, tachyons

One can see from Eqs.(50), (51), that for “bradyons” (particles on the timelike world-lines with $m^2 > 0$) the velocities and “initial” coordinates satisfy an inequality $|\mathbf{v}/c - \mathbf{r}_{0p}/R| < 1$. For “tachyons” ($m^2 < 0$) the relevant inequality is $|\mathbf{v}/c - \mathbf{r}_{0p}/R| > 1$, and “photons” ($m^2 = 0$) are moving along the “light cones” $|\mathbf{v}/c - \mathbf{r}_{0p}/R| = 1$.

Let us examine the geometry of “light cones” more carefully. The line element (45) is invariant under FL transformations (17)-(19). In these transformations the origins of coordinates coincide at the instant $t = t' = 0$. The special invariant point of transformations (17)-(19) is $t = t' = -R/c$. Let us change the time scale $t \Rightarrow t - R/c$, $t' \Rightarrow t' - R/c$. Now the origins of space coordinates coincide at the time $t = t' = R/c \equiv t_0$. We denote the velocity of k' in k by $\mathbf{u} \equiv -\mathbf{r}_0/t_0$, where $\mathbf{r}_0 = -\mathbf{u}t_0$ is the origin of coordinates of k' in k at instant $t = 0$. Eqs.(17)-(19) may be written as

$$t' = \frac{t}{\gamma_0 - (\gamma_0 - 1)t/t_0 - \gamma_0 \mathbf{r}_0 \mathbf{r}/R^2}, \quad (52)$$

$$\mathbf{r}'_{||} = \frac{\gamma_0 (\mathbf{r}_{||} + \mathbf{r}_0 t/t_0 - \mathbf{r}_0)}{\gamma_0 - (\gamma_0 - 1)t/t_0 - \gamma_0 \mathbf{r}_0 \mathbf{r}/R^2}, \quad (53)$$

$$\mathbf{r}'_{\perp} = \frac{\mathbf{r}_{\perp}}{\gamma_0 - (\gamma_0 - 1)t/t_0 - \gamma_0 \mathbf{r}_0 \mathbf{r}/R^2}, \quad (54)$$

$$(\gamma_0 \equiv 1/\sqrt{1 - r_0^2/R^2}, r_0 < R \text{ if } u < c).$$

²It is useful to note that the velocity of light at point \mathbf{r} depends on its direction and varies from $c(1 - r_{0p}/R)$ to $c(1 + r_{0p}/R)$. The velocity of the particle at some point may be more than c , but less than the velocity of light at this point in the same direction.

The metric line element (45) takes the form

$$d\hat{s}^2 = \frac{t_0^2 dt^2}{t^4} (R^2 - (\mathbf{r} - \mathbf{v}t)^2) = \frac{t_0^2 dt^2}{t^4} (R^2 - r_{0p}^2). \quad (55)$$

Here $\mathbf{r}_{0p} \equiv \mathbf{r} - \mathbf{v}t$ is an auxiliary space coordinate of the particle at the special point $t = 0$ in the case of a constant velocity \mathbf{v} .

Now the velocity four-vector (47) is

$$dx^\mu = \frac{t_0 dt}{t^2} (R, \mathbf{v}t - \mathbf{r}) = \frac{t_0 dt}{t^2} (R, -\mathbf{r}_{0p}), \quad (56)$$

and the energy-momentum four-vector (49) is $(\gamma_{0p} \equiv 1/\sqrt{1 - r_{0p}^2/R^2})$

$$p^\mu = m\gamma_{0p} \left(1, \frac{\mathbf{v}t - \mathbf{r}}{R} \right) = m\gamma_{0p} \left(1, \frac{-\mathbf{r}_{0p}}{R} \right). \quad (57)$$

Energy and momentum of the particle with mass m are now

$$E = \gamma_{0p}m = \frac{m}{\sqrt{1 - (\mathbf{r} - \mathbf{v}t)^2/R^2}}, \quad (58)$$

$$\mathbf{p} = \gamma_{0p}m \frac{-\mathbf{r}_{0p}}{R} = m \frac{\mathbf{v}t - \mathbf{r}}{R\sqrt{1 - (\mathbf{r} - \mathbf{v}t)^2/R^2}}. \quad (59)$$

At the instant $t = 0$ all bradyons are inside the sphere $\mathbf{r}^2 = R^2$ and their energy and momentum are connected with their position by the relation

$$\frac{\mathbf{r}_{0p}^2}{R^2} = 1 - \frac{m^2}{E^2} = \frac{\mathbf{p}^2}{E^2} \quad (60)$$

Their velocity is arbitrary and does not depend on energy or momentum.

In the case of a lightlike invariant (55) $d\hat{s}^2 = 0$ we see that all world lines of photons ($m^2 = 0$) are passing through the points $r_{0p}^2 = R^2$ at $t = 0$.

At those points their velocities were arbitrary. Having velocity \mathbf{c} at $t = 0$, photons are moving along the world line

$$\mathbf{r}(t, c) = \mathbf{r}_{0p} + \mathbf{c}t. \quad (61)$$

The velocity of the photons in any world point t , \mathbf{r} is equal to

$$\mathbf{c}(t, \mathbf{r}) = \frac{\mathbf{r} - \mathbf{r}_{0p}}{t}. \quad (62)$$

The photons, which pass through the origin of the space coordinates $\mathbf{r} = 0$ at the time t , have the velocity

$$\mathbf{c}(t) = \frac{-\mathbf{r}_{0p}}{t}, \quad c(t) = \frac{R}{t}. \quad (63)$$

The light cone with the vertex at the world point t_1 , r_1 is determined by the relation

$$\mathbf{r} = \mathbf{r}_{0p} + t \frac{\mathbf{r}_1 - \mathbf{r}_{0p}}{t_1}, \quad r_{0p}^2 = R^2. \quad (64)$$

The dependence of the speed of light on time and space coordinates does not contradict the constancy of c in the FL transformation (52)-(54). This transformation relates the reference frames with coincident space origins at time t_0 . The constant c in this transformation is equal to the speed of light at world point $t_0, 0$, which is $c = R/t_0$. FL transformations at other moments t will have other constant $c = R/t$. The metric line element (55) and four-vector (56) depend on t_0 , but four-vector (57) and all subsequent ones are independent on t_0 . Eqs.(50), (51) for energy and momentum contain c as though they depend on the time of the FL transformation. Nevertheless, Eqs.(58), (59) obviously demonstrate, that energy and momentum of the particle are completely determined by the straight world line $\mathbf{r} = \mathbf{r}_{0p} + \mathbf{v}t$, which is tangential to the world line of this particle.

The quantity

$$ds^2 = \frac{d\hat{s}^2}{t_0^2} = \frac{dt^2}{t^4} (R^2 - r_{0p}^2) \quad (65)$$

is invariant under FL transformations (52)-(54) for any values of t_0 \mathbf{r}_0 .³

We have introduced energy E and momentum \mathbf{p} by Eqs.(58), (59). They differ from the current quantities \hat{E} , $\hat{\mathbf{p}}$ by the dimensional multiplier:

$$\hat{E} = \gamma_{0p} mc^2 = \gamma_{0p} m \frac{R^2}{t^2}, \quad (66)$$

$$\hat{\mathbf{p}} = \gamma_{0p} mc \frac{-\mathbf{r}_{0p}}{R} = \gamma_{0p} m \frac{-\mathbf{r}_{0p}}{t}. \quad (67)$$

In our approach these quantities are not conserved in time. Apparently, in the dynamics with varying speed of light just E and \mathbf{p} from Eqs.(58) and (59) should be conserved.

³Let us notice that \mathbf{r}_0 determines the velocity \mathbf{u} of k' in k : $\mathbf{u} = -\mathbf{r}_0/t_0$, and \mathbf{r}_{0p} determines the velocity \mathbf{v} of the particle in k , i. e. $\mathbf{v} = (\mathbf{r} - \mathbf{r}_{0p})/t$.

It is interesting to note that in the limit $t_0 \rightarrow 0$, taking into account $\mathbf{r}_0 = -\mathbf{u}t_0$ and $\gamma_0 \rightarrow 1 + (\mathbf{u}t_0)^2/(2R^2)$, we get Galilean transformation from Eqs.(52)-(54)⁴:

$$t' = t, \quad \mathbf{r}' = \mathbf{r} - \mathbf{u}t. \quad (68)$$

Indeed, the speed of light is going to infinity in the limit $t_0 \rightarrow 0$.

Let us note also that the sphere $\mathbf{r}^2 \leq R^2$ at $t = 0$ is invariant under FL transformation (52)-(54). We can put for clarity $R = 1$ and get from Eqs.(52)-(54) for $t = 0$

$$t' = 0, \quad (69)$$

$$\mathbf{r}'_{||} = \frac{\mathbf{r}_{||} - \mathbf{r}_0}{1 - \mathbf{r}_0 \mathbf{r}}, \quad (70)$$

$$\mathbf{r}'_{\perp} = \frac{\mathbf{r}_{\perp} \sqrt{1 - r_0^2}}{1 - \mathbf{r}_0 \mathbf{r}}. \quad (71)$$

This transformation is equivalent to

$$1 - \mathbf{r}'^2 = (1 - \mathbf{r}^2) \frac{1 - \mathbf{r}_0^2}{(1 - \mathbf{r}_0 \mathbf{r})^2}, \quad (72)$$

$$\frac{1 - \mathbf{r}'^2}{1 + \mathbf{r}_0 \mathbf{r}'} = \frac{1 - \mathbf{r}^2}{1 - \mathbf{r}_0 \mathbf{r}}. \quad (73)$$

In new variables

$$\begin{aligned} z &= 1 - \mathbf{r}^2, \\ z' &= 1 - \mathbf{r}'^2, \\ w &= 1 - \mathbf{r} \mathbf{r}_0, \\ w' &= 1 + \mathbf{r}' \mathbf{r}_0, \end{aligned}$$

which are connected by relations $ww' = (1 - r_0^2)$, $z'/w' = z/w$, the elementary volume transforms as

$$dV' = dV \frac{(1 - r_0^2)^2}{w^4}$$

or

$$\frac{dV}{z^2} = \frac{dV'}{z'^2}.$$

⁴In article [35] some modification of the special theory of relativity was suggested in which auxiliary Galilean coordinate systems were introduced for every inertial frame of reference. It induces the localized Lorentz transformation between any two usual inertial coordinate systems.

So

$$f(r) = \frac{f(0)}{(1 - \mathbf{r}^2)^2}$$

is an invariant distribution function inside the sphere $\mathbf{r}^2 \leq 1$ at $t = 0$.

VI. Connection with the Friedmann-Lobachevsky metric

Let us rewrite the metric line element (55) as

$$ds^2 = \frac{t_0^2}{t^4} \{ R^2 dt^2 - (\mathbf{r} dt - t d\mathbf{r})^2 \}, \quad (74)$$

and single out the full square with dt :

$$ds^2 = \frac{t_0^2}{t^4} \left\{ \left[\sqrt{R^2 - r^2} dt + \frac{tr dr}{\sqrt{R^2 - r^2}} \right]^2 - \frac{t^2}{t_0^2} \left[\frac{R^2 dr^2}{R^2 - r^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right] \right\}. \quad (75)$$

After the replacement

$$t = \hat{t} \sqrt{1 - \frac{r^2}{R^2}}, \quad (76)$$

which differs from the known Milne transformation [36]

$$t = \hat{t} \sqrt{1 - \frac{u^2}{c^2}}, \quad (77)$$

by the region of definition⁵, we get the line element

$$ds^2 = \frac{R^2 t_0^2}{\hat{t}^4} \left(d\hat{t}^2 - \frac{\hat{t}^2}{R^2} d\ell^2 \right), \quad (78)$$

and curved space-like hypersektion with the metric

$$d\ell^2 = \frac{1}{1 - r^2/R^2} \left(\frac{dr^2}{1 - r^2/R^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right). \quad (79)$$

After standard replacements

$$r = \frac{\tilde{r}}{\sqrt{1 + \tilde{r}^2/R^2}},$$

or

$$r = \frac{\hat{r}}{1 + \hat{r}^2/(4R^2)},$$

⁵ Eq.(77) is definite in the future light cone and Eq.(76) is definite in all observable space-time cylinder $r < R$.

we get well known forms of Friedmann-Lobachevsky space

$$d\ell^2 = \frac{d\tilde{r}^2}{1 + \tilde{r}^2/R^2} + \tilde{r}^2(d\theta^2 + \sin^2\theta d\varphi^2) = \frac{d\hat{r}^2 + \hat{r}^2(d\theta^2 + \sin^2\theta d\varphi^2)}{(1 + \hat{r}^2/(4R^2))^2}. \quad (80)$$

Eqs.(78), (80) correspond to an open (space curvature $K = -1/R^2$) flat expanding Universe with linear scale factor \hat{t}/t_0 in conformal coordinates. In the work [37] it was shown that linear dependence of the scale factor on time is not in contradiction with modern astronomical observations [38], [39]. Nevertheless, the comparison of the above results with experiment would be possible only after investigation of the red-shift parameters. The varying speed of light may change many usual relations.

VII. Red shift

Let us investigate the red shift of the light signal from the emitter in galaxy A , which has constant velocity \mathbf{u} relative to our Galaxy. We put the space coordinate origin into our Galaxy and put $t = 0$ at the special point of FL transformations (with the infinite speed of light). Let the visual big-bang instant equal to t_* , which may differ from $t = 0$.

We observe at t_1 the signal, emitted in galaxy A at t_2 , when it was at distance $\mathbf{r}_2 = (t_2 - t_*)\mathbf{u}$ from us. The velocity of this signal is $|c_1| = R/t_1$. The moments t_1 and t_2 are related by

$$(t_2 - t_*)\mathbf{u} = (t_1 - t_2)\frac{R}{t_1},$$

or

$$t_2 = \frac{R + ut_*}{R + ut_1}t_1 \quad (81)$$

and

$$\frac{dt_2}{dt_1} = \frac{R(R + ut_*)}{(R + ut_1)^2}. \quad (82)$$

Let us connect t_2 with the emitter proper time τ_2 . From the invariance of interval (74) between the world points (t_2, \mathbf{r}_2) and $(t_2 + dt_2, \mathbf{r}_2 + \mathbf{u}dt_2)$, we get

$$\frac{d\tau_2}{\tau_2^2}\gamma = \frac{dt_2}{t_2^2}, \quad (83)$$

where

$$\gamma = \frac{1}{\sqrt{1 - u^2/c_*^2}},$$

and $c_* = R/t_*$ is the speed of light at big bang⁶.

After integrating of Eq.(83) from t_* till t_2 or directly from the FL transformation Eqs.(52), (53), we get the relation

$$\frac{t_2}{\tau_2} = (1 - \gamma^{-1}) \frac{t_2}{t_*} + \gamma^{-1} \quad (84)$$

and with the help of Eq.(81)

$$\frac{t_2}{\tau_2} = (1 - \gamma^{-1}) \frac{R + ut_*}{R + ut_1} \frac{t_1}{t_*} + \gamma^{-1} \quad (85)$$

From Eqs.(82)-(85) we get

$$z + 1 = \frac{dt_1}{d\tau_2} = \left[1 + (\gamma - 1) \frac{R + ut_*}{R + ut_1} \frac{t_1}{t_*} \right]^2 \frac{(R + ut_1)^2 \sqrt{R^2 - u^2 t_*^2}}{R(R + ut_*)}. \quad (86)$$

A more evident form of relation (86) may be written with the help of c_1 and c_*

$$z + 1 = \left(1 + \frac{u}{c_1} \right)^2 \frac{\sqrt{1 - u/c_*}}{\sqrt{1 + u/c_*}} \left[1 + (\gamma - 1) \frac{u + c_*}{u + c_1} \right]^2. \quad (87)$$

In the small z limit ($u \ll c_1$, $u < |c_*|$) we get

$$z \approx \frac{u}{c_1} \left(2 - \frac{c_1}{c_*} \right).$$

In the limit $|c_*| \rightarrow \infty$ ($|c_*| \gg c_1$, $|c_*| \gg u$, with any relation between u and c_1) Eq.(87) simplifies:

$$z + 1 = \left(1 + \frac{u}{c_1} \right)^2, \quad (88)$$

Independently from the movement of the galaxies we can get the general form of the Doppler red shift if we change in Eq.(87) the speed of light at the big bang c_* by the relation

$$\frac{1}{c_*} = \frac{1}{c_1} - \frac{r_2}{R} \left(\frac{1}{c_1} + \frac{1}{u} \right). \quad (89)$$

The last equation can be derived from $R = c_1 t_1 = c_* t_*$, $r_2 = c_1(t_1 - t_2) = u(t_2 - t_*)$. Here positive values of u mean the movement of the emitter from the observer and negative u — the movement to the observer.

⁶The values of t_* and c_* may be negative. This would mean that big-bang time t_* preceeds the special point $t = 0$ of the FL transformation. At this point galaxy A would be at distance $r_0 = -ut_*$ from our Galaxy.

Now, with the help of Eq.(89), we get instead of Eq.(87)

$$z + 1 = \frac{\sqrt{1 + \frac{u}{c_1}} \left\{ \frac{u}{c_1} \sqrt{1 - \frac{r_2}{R}} - \frac{r_2}{R} \sqrt{\left(1 + \frac{u}{c_1}\right) \left[1 - \frac{u}{c_1} + \left(1 + \frac{u}{c_1}\right) \frac{r_2}{R}\right]} \right\}^2}{\left[\frac{u}{c_1} - \left(1 + \frac{u}{c_1}\right) \frac{r_2}{R}\right]^2 \sqrt{\left(1 - \frac{r_2}{R}\right) \left[1 - \frac{u}{c_1} + \left(1 + \frac{u}{c_1}\right) \frac{r_2}{R}\right]}}. \quad (90)$$

Let us investigate some limiting cases.

1. The longitudinal Doppler effect ($r_2 \ll R$):

$$z + 1 = \sqrt{\frac{c_1 + u}{c_1 - u}}, \quad (91)$$

which is the usual form.

2. Ultrarelativistic emitter, moving to the observer, ($u \approx -c_1$):

$$z + 1 = \sqrt{\frac{R - r_2}{2R}} \sqrt{1 + \frac{u}{c_1}} \quad (92)$$

Here we have blue shift, which is enhanced for the emission from the region $r_2 \rightarrow R$.

3. Emitter with any velocity $u \neq -c_1$ in the region $r_2 \rightarrow R$

$$z + 1 = \sqrt{\frac{2R}{R - r_2}} \left(1 + \frac{u}{c_1}\right)^{3/2} \quad (93)$$

For all velocities $u \neq -c_1$ we get “geometrical” red shift from the region $r_2 \approx R$, due to the denominator in Eq.(93)

4. For emitter at rest ($u = 0$):

$$z + 1 = \sqrt{\frac{R + r_2}{R - r_2}}. \quad (94)$$

We get “distant” red shift, which has quasi-Hubble form $z \approx r_2/R$ for small distances $r_2 \ll R$.

5. The first order contributions of the velocity of the emitter $u \ll c_1$ and of the distance from it $r_2 \ll R$ to the red shift value are

$$z = \frac{u}{c_1} + \frac{r_2}{R}. \quad (95)$$

Let us connect this result with the Hubble constant $H_1 = u/r_1$. Here r_1 is the modern distance to the galaxy, which we “see” at distance r_2 . From the obvious relations $r_1 - r_2 = u(t_1 - t_2)$ and $r_2 = c_1(t_1 - t_2)$ we get $r_1 = r_2(1 + u/c_1)$ and

$$H_1 = \frac{uc_1}{r_2(c_1 + u)}. \quad (96)$$

For small z we have $H_1 \approx u/r_2$ and from Eq.(95)

$$H_1 = \frac{zc_1}{r_2} - \frac{c_1}{R}. \quad (97)$$

We see that in our approach the Hubble constant is less than $H_0 = zc_1/r_2$ in the Standard Model of expanding Universe. It appears impossible to get the Hubble constant directly from its z dependence on r_2 for nearest galaxies. We need some further assumptions. For example, if $|c_*| \gg c_1$ we have $H_1 \approx c_1/R$ or

$$H_1 = \frac{zc_1}{2r_2} \approx 30 \text{ km/c/Mpc},$$

which is two times less than in the Standard Model. The corresponding age of the Universe

$$T = \frac{R}{c_1} = H_1^{-1} \approx 33 \cdot 10^9 \text{ y}$$

is two times more than the standard one.

VIII. Conclusions

We have shown that on the basis of the principle of relativity it is possible to get a more general geometry of matter free space-time in which the speed of light depends on the time of observation. A new constant R (an invariant radius of the visible part of the Universe) appears in this approach.

This model may be seen as an alternative tool for the interpretation of different cosmological events.

The Newtonian world with the infinite value of c is a limiting point of the relativistic world. On the same basis the Einstein world with infinite value of R may be seen as a limiting point in more general approach.

If the real value of R in our world is infinitely large and the speed of light does not depend on time, we need to find some physical principle which rules out such variant of the space-time structure.

Appendix A

After this paper was completed an interesting work [40] was published. In this work the authors start from the inertial frame with global coordinate system (x^0, x^1, x^2, x^3) and look for all coordinate systems $\xi^\mu(x, v)$, such that

$$\frac{d^2\xi^\mu}{ds^2} = 0. \quad (98)$$

In view of $dx^\mu/ds = v^\mu$, and $dv^\mu/ds = 0$ they get

$$\frac{\partial^2\xi^\mu}{\partial x^\alpha\partial x^\beta}v^\alpha v^\beta = 0, \quad (99)$$

i.e. ξ^μ must be linear in x^μ . But Eq.(98) is redundant for the free motion of the particle. We can put

$$\frac{d}{ds}\left(\frac{d\xi^\mu}{ds}/\frac{d\xi^0}{ds}\right) = 0$$

instead of (98). After some complicated calculations (all details can be seen in [1]) we find

$$\frac{\partial^2(w\xi^\mu)}{\partial x^\alpha\partial x^\beta}v^\alpha v^\beta = 0$$

instead of (99), where

$$\frac{\partial^2w}{\partial x^\alpha\partial x^\beta}v^\alpha v^\beta = 0.$$

So, w must be linear, but all four functions ξ^μ may be linear fractions in x^ν with equal denominators w , as we postulated in Eqs.(1)-(4). Only if w is constant, ξ^μ are reduced to linear functions.

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